TRIVIAL CUT POINT IS NOT FAR
NON-TRIVIAL CUT POINT IS FAR

a) NOT FAR $\not\Rightarrow$ CUT POINT

$$E_n = \{ k \cdot 2^{-n} : 0 \le k \le 2^n \}$$

$$E = \bigcup_n \{n\} \times E_n$$

$$\mathcal{F} = \{ E \setminus f : f \in \prod_{n \in \omega} E_n \}$$

IF x EXTENDS $\mathcal{F}$
THEN x IS NOT FAR
ALSO NOT A CUT POINT

b) FAR $\not\Rightarrow$ CUT POINT

$$\mathcal{F} = \{ F \subseteq M : \lambda(M \setminus F) < \infty, \ F \text{ CLOSED} \}$$

IF D CLOSED DISCRETE
THEN $F \cap D = \emptyset$ FOR SOME F

$x \supseteq \mathcal{F}$
NOT CUT POINT

$a \in A_x$ $b \in B_x$

$$\sum_n (b_n - a_n) = \infty$$

$x \in M^*$ FAR
NEED TO SHOW
IN $V[G_{\omega_2}]$ THERE
IS NO $y \in M^*$ THAT
EXTENDS x AND IS
A CUT POINT OF ITS $\mathbb{I}_u$

WE NEED $f \in \omega^\omega$
SUCH THAT FOR ALL $g \in \omega^\omega$
WITH $g(n) < f(n)$

$$\bigcup_n \{n\} \times \left[\frac{g(n)}{f(n)}, \frac{g(n)+1}{f(n)} \right] \notin y$$

$T \subseteq {}^{<\omega}\omega$: $s_T \in T$

$t \in T$: $t \subseteq s_T$

$s_T \subseteq t$

$s_T \subseteq t \in T$: $\{i : t^\frown i \in T\}$

IS INFINITE

$S \leq T$: $S \subseteq T$.

G GENERIC ON $\mathbb{L}$ GIVES US

$f : \omega \to \omega$

$\{f\} = \bigcap \{[T] : T \in G\}$

$f = \bigcup \{s_T : T \in G\}$

DOMINATES ω^ω

$\mathbb{P}_{\omega_2} = $ "$\mathbb{L} * \dot{\mathbb{P}}_{\omega_2}$"

- GIVES US f
- IF g IS A FUNCTION IN $V[G_{\omega_2}]$ WITH $g(n) < f(n)$

THEN IN $V[f]$

WE CAN HAVE

$\langle F(n) : n \in \omega \rangle$

- $F(n) \subseteq f(n)$
- $|F(n)| = n+1$
- $\Vdash \dot{g}(n) \in F(n)$ FOR ALL n)

$$\left[\frac{\dot{g}(n)}{f(n)}, \frac{\dot{g}(n)+1}{f(n)}\right]$$

$$\subseteq \bigcup_{i \in F(n)} \left[\frac{i}{f(n)}, \frac{i+1}{f(n)}\right]$$

VERY VERY THIN

TECHNICAL POINT:

$$^{<\omega}\omega \longleftrightarrow \{t \in T : s_T \le t\}$$

HAS AN ORDER IN TYPE ω WITH TWO PROPERTIES

- $s \subset t \Rightarrow s \prec t$
- $s^\frown m \prec s^\frown n$ IF $m < n$

$$\{t \in T : s_T \le t\} = \{T\langle 0\rangle, T\langle 1\rangle, T\langle 2\rangle, \dots\}$$

LAVER'S NOTATION

$S \le^n T$ MEAN

- $S \le T$
- $S\langle i\rangle = T\langle i\rangle$ $i \le n$

$T\langle 0\rangle = s_T$ $\qquad$ $S\langle 0\rangle = s_S$

IF $(T_m)_m$ IS ~~A~~ SEQUENCE SUCH THAT FOR ALL m

$$T_{m+1} \le^m T_m$$

$T_m\langle 0\rangle, \dots, T_m\langle m\rangle$ WILL SURVIVE

THEN $\bigcap_m T_m \in \mathbb{L}$ $\qquad$ $T_\omega \le^m T_m$

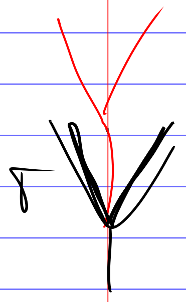
$$T_\omega = \{T_m\langle m\rangle : m \in \omega\}$$

$T \in \mathbb{L}$, $X \in V$ FINITE

$$T \Vdash \dot{x} \in X$$

THEN THERE ARE $a \in X$ AND $S \le^0 T$ SUCH THAT

$$S \Vdash \dot{x} = a$$



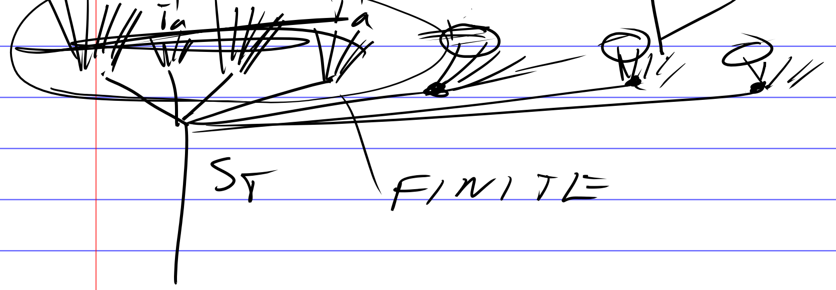
SUPPOSE NOT

THERE IS NO $S \le^\circ T$
WITH AN $a \in X$
SUCH THAT $S \Vdash \dot{x} = a$

LOOK AT

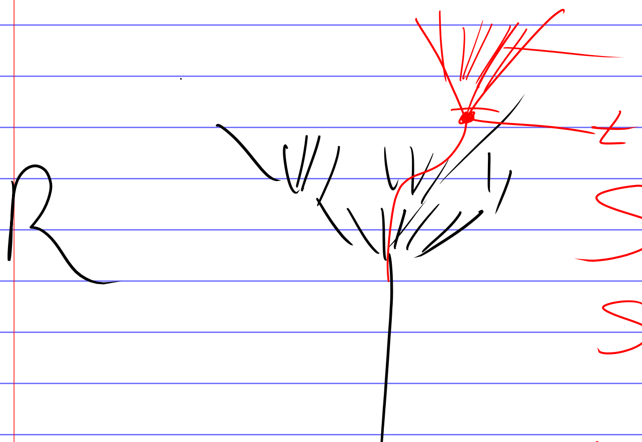
$A = \{ n : S_T{}^\frown n \in T$

AND THERE ARE
$T' \le^\circ T_{S_T{}^\frown n}$ AND a
SUCH THAT $T' \Vdash \dot{x} = a \}$



IF A WERE INFINITE
THEN WE COULD
TAKE $B \subseteq A$ INFINITE
WITH THE SAME $a \in X$
FOR ALL $n \in B$

UNION OF T's : GIVES US S
$S \Vdash \dot{x} = a$



FORCES A VALUE ON $\dot{x}$

$S \le^\circ R_t$

$S \Vdash \dot{x} = a$

CONTRADICTION.

GENERALLY

$T \in \mathbb{L}$, $X \in V$ FINITE

$$T \Vdash \dot{x} \in X$$

FOR EVERY n THERE IS AN $S \leq^n T$ AND $F \subseteq X$ WITH $|F| \leq n+1$

$$S \Vdash \dot{x} \in F.$$

APPLICATION.

WE HAVE $\dot{g} : \omega \to \omega$, $\dot{g}(n) < f(n)$ ALL n

$T \Vdash \dot{g}(n) < f(n)$

$T_0 \leq^0 T$ AND ONE VALUE k_0

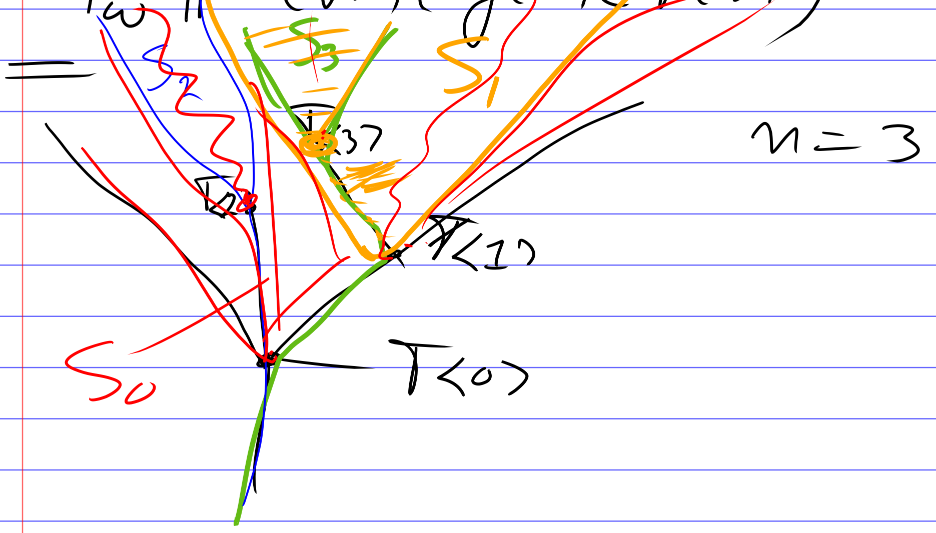
$T_0 \Vdash \dot{g}(0) = k_0 < f(0)$

$T_1 \leq^1 T_0$ AND $F(1) \subseteq f(1)$, SIZE 2

$T_1 \Vdash \dot{g}(1) \in F(1)$

$T_2 \Vdash \dot{g}(2) \in F(2)$ $\quad |F(2)| = 3$

$T_\omega \Vdash (\forall n)(\dot{g}(n) \in F(n))$



IF $T \Vdash \dot{x} \in V$

THEN THERE IS $S \leq^{0} T$ AND A COUNTABLE $A \subset V$

$S \Vdash \dot{x} \in A$

SAME THING FOR THE ITERATION

LEMMA 6 OF LAVER

$\mathbb{P}_{\omega_2} \ni p$; $F \subseteq \omega_2$ FINITE

$n \in \omega$

$X \in V$ FINITE $\quad p \Vdash \dot{x} \in X$

THERE ARE $q \in \mathbb{P}_{\omega_2}$ AND $G \subseteq X$

SUCH THAT $\quad$ – $|G| \leq (n+1)^{|F|}$

– $q \Vdash \dot{x} \in G$

– $q \leq^{n}_{F} p$

– $q \leq p$

– $\alpha \in F$: $q \restriction \alpha \Vdash q(\alpha) \leq^{n} p(\alpha)$

$q_{n+1}(\alpha) \leq^{n} q_n(\alpha)$

$\bigcup_n F_n = \bigcup_n \operatorname{supp}(q_n)$

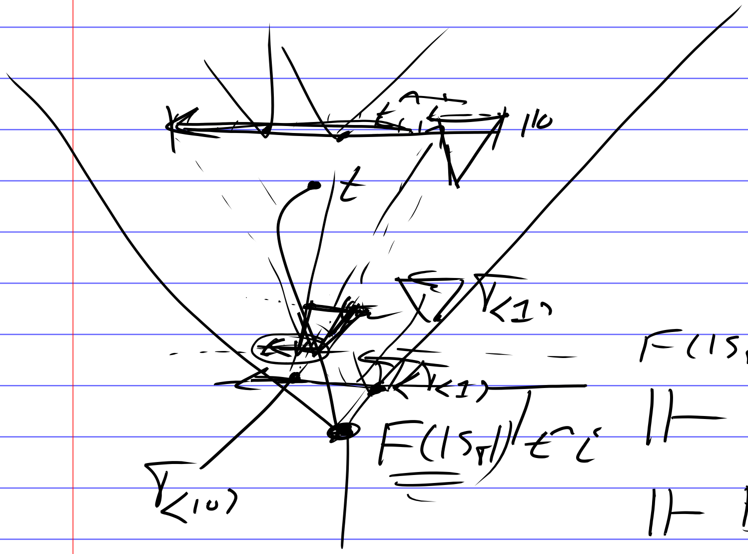
$(n+1)^{n+1}$

$V \quad V[f] \quad V[G_{\omega_2}]$

$\dot{f} \quad F \underset{\leq n}{\text{———}} g$

$\dot{F}$

$2^{\aleph}$



$\dot{f}$

$\dot{F}$

$F(|s_T|+1)$

$\Vdash \dot{f}(|t|) = i$

$\Vdash \dot{F}(|t|) \in i$

$[i]^{|t|}$ IS FINITE

- $S \leq^0 T_{t^\frown i}$
- $H(t,i) \in i$

$S \Vdash \dot{F}(|t|) = H(t,i)$

$$x_t \xleftarrow{\in} \mathbb{I}^{|t|} \longleftarrow \prod_j \left[\frac{j}{i}, \frac{j+1}{i}\right] \subseteq I^{|t|}$$

$\frac{j}{i}$ $\left[\frac{j}{i}, \frac{j+1}{i}\right]$