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$2^{\aleph_0}$ CAN BE ANYTHING IT OUGHT TO BE

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We consider countable models for Zermelo–Fraenkel set theory (including the axiom of choice), which we call, for brevity, ZF*-models. Let $\mathfrak{M}$ $(=(M, \in_M))$ and $\mathfrak{N}$ be ZF*-models. Then $\mathfrak{N}$ is an *excellent extension* of $\mathfrak{M}$ if and only if: (a) $\mathfrak{M}$ is a complete submodel of $\mathfrak{N}$; (b) the ordinals (cardinals) of $\mathfrak{N}$ are exactly the ordinals (cardinals) of $\mathfrak{M}$; and (c) the confinality, $\text{cf}(\aleph)$, of a cardinal $\aleph$ has the same value in $\mathfrak{M}$ as in $\mathfrak{N}$.

The proofs of the following three theorems use the new techniques introduced in Cohen [63, 64]. (The first theorem is due, independently, to Cohen.) Let $\mathfrak{M}$ be a ZF*-model in which $V=L$ is valid, fixed once for all.

Theorem 1. *Let $\aleph$ be an infinite cardinal of $\mathfrak{M}$ with $\aleph_0<\text{cf}(\aleph)$. Then there is an excellent extension $\mathfrak{N}$ of $\mathfrak{M}$ in which $2^{\aleph_0}=\aleph$.*

Theorem 2. *Let $\aleph$ and $\aleph'$ be infinite cardinals of $\mathfrak{M}$ with $\aleph=\text{cf}(\aleph)<$ $<\text{cf}(\aleph')$. Then there is an excellent extension $\mathfrak{N}$ of $\mathfrak{M}$ in which:*

(i) $2^{\aleph}=\aleph'$;
(ii) if $\aleph_\alpha<\aleph$, then $2^{\aleph_\alpha}=\aleph_{\alpha+1}$.

Theorem 3. *Identify the ordinary integers with an initial segment of the integers of $\mathfrak{M}$. Let $k, n_0, \ldots, n_k$ be ordinary integers and suppose that $i<n_i$ (for $0\leqslant i\leqslant k$) and $n_0\leqslant n_1\leqslant \ldots \leqslant n_k$. Then there is an excellent extension $\mathfrak{N}$ of $\mathfrak{M}$ in which $2^{\aleph_i}=\aleph_{n_i}$ (for $0\leqslant i\leqslant k$).*

Remarks on the proof Theorem 2. The essential point is to make sure that no "new" sets of cardinality less than $\aleph$ land in $\mathfrak{N}$. This is insured as follows: (1) we add a generic subset of $\aleph'$, say A; (2) a set of conditions on A will be a set (of $\mathfrak{M}$) of conditions of the form "$\alpha \in A$" (or "$\neg\, \alpha \in A$") whose *cardinality in $\mathfrak{M}$ is less than $\aleph$*. (Of course, outside of $\mathfrak{M}$, the sets of conditions are denumerable since $\mathfrak{M}$ is, so Cohen's diagonal construction applies.) The crucial observation is the following:

Lemma. *Let Σ be a set (in $\mathfrak{M}$) of limited statements whose cardinality (in $\mathfrak{M}$) is less than $\aleph$. Then any set of conditions P has an extension forcing each member of Σ.*