List of errata

April 6, 2010

1. Page 4, line 2:
$$\frac{1}{\Gamma(x)} = \lim_{n\to\infty} \frac{x(x+1)\cdots(x+n)}{n!\,n^x}$$
   Note that (1.1.5) is equivalent to $\Gamma(x) = \lim_{k\to\infty} \dfrac{k!\,k^x}{(x)_{k+1}}$ (compare with page 35).

2. Page 11, line 15: "$x\csc\pi x$" should read "$\pi\csc\pi x$".

3. Page 21, line –6 (delete the comma in $\frac{\pi}{2}$):
$$\arctan\frac{b}{a} + \arctan\frac{a}{b} = \begin{cases} \frac{\pi}{2}, & \text{if } b > 0, \\ -\frac{\pi}{2}, & \text{if } b < 0. \end{cases}$$

4. Page 21, line –4: "$O\left(\dfrac{1}{b^2}\right)$" should read "$O\left(\dfrac{1}{b^3}\right)$".

5. Page 23, line 2: "Guass's" should read "Gauss's".

6. Page 23, Remark 1.5.1: "verifed" should read "verified".

7. Page 48, Exercise 13 (a) (i) should read:
   Integration by parts gives $C(x, y+1) = \frac{x}{y} C(x+1, y)$.

8. Page 50, Exercise 17. "$a > 0$" should read "$a > 0$ and $b > 0$".

9. Page 62, Proof of Theorem 2.1.1. "$p < q$" should read "$p < q+1$" or "$p \le q$".

10. Page 62, line -2: "coefficient of nth term" should read "coefficient of the nth term".

11. Page 82, line 10: this should read (remove x^{-n})
$$k x^\alpha \sum_{n=-\infty}^{\infty} \frac{(1-c-\alpha)_n(-\alpha)_n}{(1-a-\alpha)_n(1-b-\alpha)_n} x^{-n}.$$

12. Page 83, line 8: the minus sign should be a plus sign.

13. Page 85, after (2.4.1):
    Take a rectangular contour L with vertices $c \pm iR$, $-(N+\frac{1}{2}) \pm iR$, where N is a positive integer (delete "c" in "$c - (N+\frac{1}{2}) \pm iR$").

14. Page 85, line –4: "$F(x)$" should read "$F(s)$".

15. Page 89, formula (2.4.10) should read
$$\frac{1}{2\pi i} \int_{k-i\infty}^{k+i\infty} \frac{\Gamma(a+s)\Gamma(1+c-s)}{\Gamma(b+s)\Gamma(1+d-s)} \, ds = \frac{\Gamma(a+c)\Gamma(b+d-a-c-1)}{\Gamma(b-a)\Gamma(d-c)\Gamma(b+d-1)}.$$

16. Page 95, formula (2.5.4) should read (insert a factor x to the last term):

$$(1-x)_2F_1\left(\begin{matrix}a,b\\c\end{matrix};x\right) = \frac{c-(2c-a-b+1)x}{c}{}_2F_1\left(\begin{matrix}a,b\\c+1\end{matrix};x\right)$$
$$+ \frac{(c-a+1)(c-b+1)}{c(c+1)}x\,{}_2F_1\left(\begin{matrix}a,b\\c+2\end{matrix};x\right).$$

17. Page 101, 4 lines below (2.5.17):

It is not difficult to show that $T_n(x) = C\,P_n^{(-1/2,-1/2)}(x)$, where $C = 2^{2n}(n!)^2/(2n)!$.

In view of Exercise 19, the connection between $U_n(x)$ and the Jacobi polynomials should also be mentioned here; compare with (5.1.1) and (5.1.2).

18. Page 115, Exercise 4 (a) should read:

Show that $\frac{1}{2}\left((1+x)^n + (1-x)^n\right) = {}_2F_1(-n/2, -(n-1)/2; 1/2; x^2)$.

19. Page 117, Exercise 19 (c). There should be a reference to (5.1.1) and (5.1.2) here.

20. Page 118, Exercise 22:

$$2(n+1)(n+\alpha+\beta+1)(2n+\alpha+\beta)P_{n+1}^{(\alpha,\beta)}(x)$$
$$= (2n+\alpha+\beta+1)\left\{(2n+\alpha+\beta)(2n+\alpha+\beta+2)x+\alpha^2-\beta^2\right\}P_n^{(\alpha,\beta)}(x)$$
$$- 2(n+\alpha)(n+\beta)(2n+\alpha+\beta+2)P_{n-1}^{(\alpha,\beta)}(x), \quad n=1,2,3,\ldots$$

(delete " $= 0$").

21. Page 118, Exercise 24 (d). This contiguous relation is wrong. It can be replaced by

$$c\left(b-(c-a)x\right)F - bc(1-x)F(b+) + (c-a)(c-b)xF(c+) = 0.$$

A contiguous relation between F and $F(b\pm)$ is (compare with (a))

$$(c-2b+(b-a)x)F + b(1-x)F(b+) + (b-c)F(b-) = 0.$$

22. Page 123, Exercise 44. "Theorem 2.2.1" should read "Theorem 2.2.2".

23. Page 144, line 13: ${}_2F_1\left(\begin{matrix}a,b;-1\\a-b+1\end{matrix}\right)$ should read ${}_2F_1\left(\begin{matrix}a,b\\a-b+1\end{matrix};-1\right)$.

24. Page 189, line 3: there is some redundant extra space in the second solution. So

$$x^{1-c}{}_2F_1\left(\begin{matrix}a+1-c,b+1-c\\2-c\end{matrix};x\right) \text{ should read } x^{1-c}{}_2F_1\left(\begin{matrix}a+1-c,b+1-c\\2-c\end{matrix};x\right).$$

25. Page 191, (4.1.11) should read:

$$_1F_1\left(\begin{matrix}a\\c\end{matrix};x\right) = e^x\,{}_1F_1\left(\begin{matrix}c-a\\c\end{matrix};-x\right).$$

26. Page 196, line -1: It is easy to see that $\operatorname{erf} x = \frac{2x}{\sqrt{\pi}}{}_1F_1\left(1/2; 3/2; -x^2\right)$.

27. Page 197, (4.4.6) should read:

$$\gamma(a, x) = \frac{x^a}{a} \, {}_1F_1 \left(a; a + 1; -x\right).$$

28. Page 199, line –2: "$y(x) = \sqrt{x} \, W(2ix)$" should read "$y(x) = W(2ix)/\sqrt{x}$".

29. Page 200, in (4.5.7): "$\ln \frac{x}{2}$" should read "$\log \frac{x}{2}$".

30. Page 201, below (4.5.9): "$x = 1/2$" should read "$a = 1/2$".

31. Page 201, below (4.5.9): "$\alpha^2 = 1/a$" should read "$\alpha = a/c = 1/3$".

32. Page 204, (4.7.5) should read:

$$J_\alpha(x) = \frac{1}{\sqrt{\pi} \, \Gamma(\alpha + 1/2)} \, (x/2)^\alpha \int_{-1}^{1} e^{ixt} (1 - t^2)^{\alpha - 1/2} \, dt$$

("dt" is missing).

33. Page 204, (4.7.7): there should be a reference to the definition (6.4.9) of the ultraspherical or Gegenbauer polynomials.

34. Page 205, last line above Figure 4.1: this should read (insert 2^α) $= \dfrac{2\pi i \sqrt{\pi}}{\Gamma(\frac{1}{2} - \alpha)} 2^\alpha J_\alpha(x).$

35. Page 208, (4.7.21) should read:

$$H_{-1/2}^{(1)}(x) = \sqrt{\frac{2}{\pi x}} \left(\cos x + i \sin x\right) = \sqrt{\frac{2}{\pi x}} \, e^{ix}, \quad x > 0$$

(delete "$= H_{1/2}^{(2)}(x)$").

36. Page 209, (4.7.22) should read:

$$H_{-1/2}^{(2)}(x) = \sqrt{\frac{2}{\pi x}} \left(\cos x - i \sin x\right) = \sqrt{\frac{2}{\pi x}} \, e^{-ix}, \quad x > 0.$$

This is not equal to $H_{1/2}^{(1)}(x)$.

37. Page 213, line 2: this should read (first sum over m instead of n)

$$1 = \sum_{m=-\infty}^{\infty} J_m(x) t^m \sum_{n=-\infty}^{\infty} (-1)^n J_n(x) t^n.$$

38. Page 213, line –1: this should read (y instead of x in the last sum)

$$\sum_{n=-\infty}^{\infty} J_n(x + y) t^n = \sum_{m=-\infty}^{\infty} J_m(x) t^m \sum_{n=-\infty}^{\infty} J_n(y) t^n.$$

39. Page 215, line 4: this should read (divide by ab in the right-hand side)

$$\frac{J_1(c)}{c} = 2 \sum_{m=1}^{\infty} m \frac{J_m(a)}{a} \frac{J_m(b)}{b} \frac{\sin m\theta}{\sin \theta}.$$

40. Page 216, line 6: this should read ($e^{im\theta}$ instead of $e^{in\theta}$)

$$= \int_0^{\infty} \frac{1}{2\pi} \int_0^{2\pi} \sum_{m=-\infty}^{\infty} i^m J_m(Rr) e^{im\theta} \sum_{n=-\infty}^{\infty} f_n(r) e^{in(\theta+\phi)} \, d\theta \, r \, dr.$$

41. Page 219, (4.11.13) can just be written as (for $\mathrm{Re}\,(x \pm iy) > 0$)

$$\int_0^{\infty} e^{-xt} J_0(yt) \, dt = \frac{1}{\sqrt{x^2 + y^2}}.$$

Delete the brackets in $\sqrt{(x^2 + y^2)}$.

42. Page 223, (4.12.5) can also be written as

$$K_{1/2}(x) = K_{-1/2}(x) = \sqrt{\frac{\pi}{2x}}\, e^{-x}, \quad x > 0.$$

43. Page 223, (4.13.1) can just be written as (for $\mathrm{Re}\,x > 1$)

$$\int_0^{\infty} e^{-xt} I_\alpha(t) \, dt = \frac{\left[x - \sqrt{x^2 - 1} \right]^{\alpha}}{\sqrt{x^2 - 1}}.$$

Delete the brackets in $\sqrt{(x^2 - 1)}$ (twice).

44. Page 226, (4.14.2) should read

$$\left(b^2 - a^2 \right) \int_0^x t J_\alpha(at) J_\alpha(bt) \, dt = x \left[a J_\alpha(bx) J_\alpha'(ax) - b J_\alpha(ax) J_\alpha'(bx) \right].$$

45. Page 227, line 2:

$$\frac{d}{dx} \left[ax J_\alpha(bx) J_\alpha'(ax) - bx J_\alpha(ax) J_\alpha'(bx) \right] = \left(b^2 - a^2 \right) x J_\alpha(ax) J_\alpha(bx).$$

46. Page 237, Exercise 16. The last sentence should read: Deduce that

$$J_\alpha(x) J_\beta(x) = \frac{(x/2)^{\alpha+\beta}}{\Gamma(\alpha+1)\Gamma(\beta+1)} \sum_{n=0}^{\infty} \frac{(-1)^n (\alpha+\beta+1)_{2n}}{(\alpha+1)_n (\beta+1)_n (\alpha+\beta+1)_n \, n!} \left(\frac{x}{2} \right)^{2n}.$$

47. Page 239, Exercise 29. "where $u = (a - be^{-i\theta})/c$" should read "where $u = (a - be^{-i\theta})/c$ and $c^2 = a^2 + b^2 - 2ab\cos\theta$".

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48. Page 244, Definition 5.2.1 should read: *We say that a sequence of polynomials $\{p_n(x)\}_0^\infty$, where $p_n(x)$ has exact degree n, is orthogonal with respect to the distribution $d\alpha(x)$ on the interval $[a,b]$, if*

$$\int_a^b p_n(x)p_m(x)\,d\alpha(x) = h_n\delta_{mn}.$$

("on the interval $[a,b]$" is missing.)

49. Page 244, in Theorem 5.2.2: the choice $k_n > 0$ is unnecessary. This choice effects that Corollary 5.2.6 on page 247 is correct. However, for $p_n(x) = L_n^{(\alpha)}(x)$ we have

$$p'_{n+1}(x)p_n(x) - p_{n+1}(x)p'_n(x) < 0 \quad \text{for all } x.$$

Note that Corollary 5.2.6 is used in the proof of Theorem 5.4.2 on page 253.

50. Page 251, below (5.3.9): "Since (5.3.9) is true for deg $Q(x) \le n - 1, \ldots$".
So: "(5.3.9)" instead of "(5.3.8)".

51. Page 253, (5.4.2) should read:

$$\int_a^b Q(x)\,d\alpha(x) = 0.$$

52. Page 254, line 6 should read (λ_j is missing):

$$\int_a^b g(x)P_m(x)\,d\alpha(x) = \sum_{j=1}^n \lambda_j g(x_{jn})P_m(x_{jn}).$$

53. Page 271, Exercise 8 (a): "$T_n(\cos\theta) = \cos n\theta$ and $U_n(\cos\theta) = \sin n\theta/\sin\theta$" should read "$T_n(\cos\theta) = \cos n\theta$ and $U_n(\cos\theta) = \sin(n+1)\theta/\sin\theta$".

54. Page, 273, Exercise 15. At this stage the Laguerre and Hermite polynomials are not yet defined properly. For (a) one needs (6.2.2) and (6.2.3) for instance and for (b) one needs (6.1.3) and (6.1.5) for instance. In (a) we also have $L_0^\alpha(x) = 1$ and $L_{-1}^\alpha(x) = 0$. Furthermore, in (a) "$L_n^\alpha(\ x\ )$" should read "$L_n^\alpha(x)$". And instead of "$n = 0, 1, 2, 3, \ldots$" it would be more consistent to write "$n = 0, 1, 2, \ldots$".

55. Page 279, below (6.1.8): This can be obtained by writing

$$e^{2xr-r^2} = \sum_{p=0}^\infty \frac{(2x)^p}{p!}\,r^p \sum_{q=0}^\infty \frac{(-1)^q r^{2q}}{q!}$$

and equating the coefficient of r^n on each side.

56. Page 281, line 4 should read

$$\int_{-\infty}^\infty e^{-a^2x^2-2bx}\,dx = \sqrt{\frac{\pi}{a^2}}\,e^{b^2/a^2}, \quad a \ne 0.$$

57. Page 283, line -1 should read

$$(1-r)^2 \frac{\partial F}{\partial r} + [x - (\alpha+1)(1-r)] F = 0.$$

58. Page 288, line 5: this should read (insert a factor t^α)

$$\int_0^\infty L_n^\alpha(t) \, t^\alpha \left[\frac{1}{\Gamma(\beta-\alpha)} \int_t^\infty L_m^\beta(x)(x-t)^{\beta-\alpha-1} e^{-x} \, dx - L_m^\alpha(t) e^{-t} \right] dt = 0.$$

59. Page 291, (6.2.35). In order to be consistent, this should read:

$$L_n^{\alpha+\beta+1}(x+y) = \sum_{k=0}^n L_k^\alpha(x) L_{n-k}^\beta(y).$$

Compare with Exercise 16 on page 341.

60. Page 299, (6.4.4):

$$F(x,r) = 2^{\alpha+\beta} R^{-1} (1-r+R)^{-\alpha} (1+r+R)^{-\beta} = \sum_{n=0}^\infty P_n^{(\alpha,\beta)}(x) r^n.$$

61. Page 316, line -2: "$P^{(-1/2,-1/2)}(x)$" should read "$P_n^{(-1/2,-1/2)}(x)$".

62. Page 324, line -7 and further: "$He_n(x)$" should read "$He_n(x)$".

63. Page 326, line 6:
$$He'_m(x) = \frac{d}{dx} He_m(x) = m He_{m-1}(x).$$

64. Page 341, Exercise 13 should read: Show that

$$\int_0^\infty e^{-st} t^\alpha L_n^\alpha(t) \, dt = \frac{\Gamma(n+\alpha+1)}{n!} \frac{(s-1)^n}{s^{n+\alpha+1}}, \quad s > 0.$$

It is also true for $\operatorname{Re} s > 0$.

65. Page 341, Exercise 14 should read:

$$L_n^\alpha(x) = \frac{(-1)^n \Gamma(n+\alpha+1)}{\sqrt{\pi} \, \Gamma(\alpha+1/2)(2n)!} \int_{-1}^1 (1-t^2)^{\alpha-1/2} H_{2n}(t\sqrt{x}) \, dt, \quad \alpha > -1/2.$$

66. Page 341, Exercise 15. Here we have: $L_n(x) := L_n^0(x)$.

67. Page 341, Exercise 17. The second formula should read

$$\frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)} \int_0^1 t^\alpha \frac{L_n^\alpha(xt)}{L_n^\alpha(0)} (1-t)^\beta \frac{L_m^\beta(x(1-t))}{L_m^\beta(0)} \, dt = \frac{L_{n+m}^{\alpha+\beta+1}(x)}{L_{n+m}^{\alpha+\beta+1}(0)}.$$

In view of the constraints, it is better to rephrase the exercise as follows:
For $\operatorname{Re}\alpha > -1$ and $\operatorname{Re}\beta > -1$, prove that

$$\int_0^1 t^\alpha (1-t)^\beta L_n^\alpha(xt)\, dt = \frac{\Gamma(\beta+1)\Gamma(n+\alpha+1)}{\Gamma(n+\alpha+\beta+2)} L_n^{\alpha+\beta+1}(x)$$

and

$$\int_0^1 t^\alpha \frac{L_n^\alpha(xt)}{L_n^\alpha(0)} (1-t)^\beta \frac{L_m^\beta(x(1-t))}{L_m^\beta(0)}\, dt = \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{\Gamma(\alpha+\beta+2)} \frac{L_{n+m}^{\alpha+\beta+1}(x)}{L_{n+m}^{\alpha+\beta+1}(0)}.$$

68. Page 342, Exercise 20. Part (b) should read

$$\sum_{k=0}^n \binom{n}{k} H_k(x)(2y)^{n-k} = H_n(x+y).$$

69. Page 342, Exercise 21. The second formula should read (for $\lambda > -1/2$)

$$\sum_{n=0}^\infty \frac{C_n^\lambda(x)}{C_n^\lambda(1)} \frac{r^n}{n!} = 2^{\lambda-1/2}\Gamma(\lambda+1/2)e^{xr}\left(r\sqrt{1-x^2}\right)^{-\lambda+1/2} J_{\lambda-1/2}\left(r\sqrt{1-x^2}\right).$$

70. Page 365, line –1: with $L_n(x) := L_n^0(x)$.

71. Page 607, line –2: the first ”=” should be ”−”.

72. Page 611, line –1: ”$1\cdots 3\cdot(2k-1)$” should read ”$1\cdot 3\cdots(2k-1)$”.

73. Page 616, Exercise 3. This should read: ”Show that

$$e^x x^{-a} \int_x^\infty e^{-t} t^{a-1}\, dt \sim \frac{1}{x} + \frac{a-1}{x^2} + \frac{(a-1)(a-2)}{x^3} + \ldots.$$”

74. Page 648, line 1: ”[1994]” should read ”[1998]”.

75. Page 648, line 3: ”no. 98-117” should read ”no. 98-17”.

76. Page 660, ”Gauss quardrature formula” should read ”Gauss quadrature formula”.

77. Page 660, ”Harr measure” should read ”Haar Measure”.

78. Page 663, line –1. ”$H_{e_m}(x)$, 324” should read ”$He_m(x)$, 324”.

79. Page 663. The symbol $L_n(x)$ with reference to page 341 and/or 365 is missing.