Dynamics of Infinitely Spatially Complex Solutions of PDE

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I consider parabolic two-component systems of the form

$$\partial_t u = \partial_x^2 u - F'(u)$$

on the whole real axis $-\infty < x < \infty$ where $u = (u_1, u_2) \in \mathbb{R}^2$, $F(u)$ is a given smooth non-negative potential which has differential $F'(u)$ which is Lipschitz in u. Bounded for all x, t solutions are considered. Such solutions include in particular x-periodic solutions.

Domains in the functional space of initial data which are invariant with respect to dynamics generated by this equation are described in terms of spatial behavior of solutions. Invariant sets are labeled by two parameters: by the value of the Hamiltonian $|\partial_x u|^2/2 - F(u)$ (which is approximately constant on the solutions) and by the homotopy class. It turns out that the spatial behavior of solutions can be very complex, it is related to existence of minimal cycles of corresponding Jacobian metric and the complexity can be described in terms of fundamental group of related topological space.