On Strong Comparison Principles for positive solutions of the Dirichlet p-Laplacian

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A *strong comparison principle* for weak solutions $u \in W_0^{1,p}(\Omega)$ of the following degenerate or singular Dirichlet problem is proved,

$$(P) \qquad -\operatorname{div}(|\nabla u|^{p-2}\nabla u) = \lambda |u|^{p-2}u + f(x) \ \text{ in } \Omega; \quad u = 0 \ \text{ on } \partial\Omega.$$

Here, $p \in (1, \infty)$ is a given number, $\Omega \subset \mathbb{R}^N$ is a bounded domain with a connected $C^{2,\alpha}$-boundary, for some $\alpha \in (0,1)$, λ is a constant, and $0 \leq f \in L^\infty(\Omega)$. It is assumed that $0 \leq \lambda < \lambda_1$, where λ_1 denotes the first eigenvalue associated with the corresponding eigenvalue problem. The following result is proved: Let $u, v \in W_0^{1,p}(\Omega)$ be any weak solutions of (P) corresponding to f and g (in place of f), respectively, where $0 \leq f \leq g$ and $f \not\equiv g$ in Ω. Then $0 \leq u < v$ in Ω, and $\partial v/\partial\nu < \partial u/\partial\nu \leq 0$ on $\partial\Omega$ holds for the outer normal derivatives. We also investigate the case $\lambda < 0$.

AMS subject classifications: Primary 35B50, 47H07; Secondary 35J70, 47H05

Key words: Strong comparison principle; positive weak solution; p-Laplacian; first eigenvalue; divergence theorem