Scalar curvature type equations on compact riemannian manifolds

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Taking (M, g) a compact riemannian n-manifold, a, f and h three smooth functions on M and $q \in (1, \frac{n+2}{n-2})$, we study the following problem $(\mathcal{P})$

$$\Delta_g u + au = f u^{\frac{n+2}{n-2}} + h u^q$$

$$u \in C^\infty(M) \quad , \quad u > 0 \text{ on } M$$

where Δ_g is the laplacian with the minus sign convention. In the first part, we want to find some conditions on a, f and h for $\mathcal{P}$ to have a solution; we distinguish two cases ($\Delta_g + a$ is coercive and $\Delta_g + a$ is not coercive) : we give, in each case, some existence results. The second part is devoted to uniqueness problem. Finally, in the third part, we give some existence results in the case where the exponent $\frac{n+2}{n-2}$ is replaced by $p > \frac{n+2}{n-2}$ (super-critical case).