Slow motion for Burgers' type equation*

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We study for small positive ε the slow motion of the solution for evolution equations of Burgers' type with small diffusion,

$$u_t = \varepsilon u_{xx} + f(u)\, u_x, \quad u(x,0) = u_o(x), \quad u(\pm 1, t) = \pm 1, \tag{$\star$}$$

on the bounded spatial domain $[-1\,,\,1]$; f is a smooth function satisfying

$$f(1) > 0, f(-1) < 0 \quad \text{and} \quad \int_{-1}^{1} f(t)dt = 0\,.$$

The initial and boundary value problem ($\star$) has a unique asymptotically stable equilibrium solution that attracts all solutions starting with continuous initial data u_o.

On the infinite spatial domain $\mathbb{R}$ the differential equation has slow speed travelling wave solutions generated by profiles that satisfy the boundary conditions of ($\star$). As long as its zero stays inside the interval $[-1\,,\,1]$, such a travelling wave suitably describes the slow long term behaviour of the solution of ($\star$) and its speed characterises the local velocity of the slow motion with exponential precision. A solution, that starts near a travelling wave, moves in a small neighbourhood of the travelling wave with exponentially slow velocity (measured as the speed of the unique zero) during an exponentially long time interval $(0, T)$. In this paper we give a unified treatment of the problem, using both Hilbert space and maximum principle methods, and we give rigorous proofs of convergence of the solution and of the asymptotic estimate of the velocity.

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