Asymptotic behavior of weak solutions of the drift diffusion equations for semiconductors coupled with Maxwell's equations

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In this talk the transient drift-diffusion model describing the charge transport in semiconductors is considered.

$$\partial_t \rho_k(t,x) = -\nabla \cdot \mathbf{j}_k(t,x) - R(x, \rho_1(t,x), \rho_2(t,x)), \tag{1}$$

$$\mathbf{j}_k(t,x) = -D_k(x)\nabla \rho_k(t,x) - (-1)^k \mu_k(x)\rho_k(t,x)\mathbf{E}(t,x) \tag{2}$$

for $k \in \{1,2\}$

$$\varepsilon(x)\partial_t \mathbf{E}(t,x) = \text{ curl } \mathbf{H}(t,x) + \mathbf{j}_2(t,x) - \mathbf{j}_1(t,x) \tag{3}$$

$$\mu(x)\partial_t \mathbf{H}(t,x) = - \text{ curl } \mathbf{E}(t,x) \tag{4}$$

$$\text{div } (\varepsilon(x)\mathbf{E}(t,x)) = \rho_1(t,x) - \rho_2(t,x) + C(x), \text{ div } (\mu(x)\mathbf{H}(t,x)) = 0 \tag{5}$$

Here ρ_1, ρ_2 denote the charge densities and $\mathbf{j_1}, \mathbf{j_2}$ denote the current densities of the holes and electrons respectively.
The self-consistent electromagnetic field $(\mathbf{E}, \mathbf{H})$ obeys Maxwell's equations 3, 4 and 5.
The unknown functions $\rho_1, \rho_2, \mathbf{E}, \mathbf{H}$ depend on the time $t \in I\!R$ and space variable $x \in \Omega$
$\Omega \subset I\!R^3$ is a bounded Lipschitz-domain with $\partial\Omega = \Gamma_D \cup \Gamma_N$, where Γ_D, Γ_N are disjoint subsets of $\partial\Omega$.
Γ_D represents the perfectly conducting Ohmic contacts and Γ_N represents the insulating boundary of the semiconductor device. The mobilities μ_1, μ_2 of the holes and electrons resp. are assumed to be positive constants. The diffusion coefficients $D_1, D_2$and the recombination generation rate R are functions of the densities ρ_1, ρ_2 and the space variable x.
The system 2 - 5 is supplemented by suitable physically motivated initial boundary conditions.
Analysis of the drift diffusion model for semiconductors has been presented in [1] - [9] and [11]-[14] in the case that Maxwell's equations 3 - 5 are replaced by Poisson's equation $-$ div $(\varepsilon \nabla V) = \rho_1 - \rho_2 - C$ for an electrostatic field $\mathbf{E} = -\nabla V$.
However, at very high frequencies the time dependent magnetic field generates a non curl-free electric field, which cannot be written as the gradient of an electrostatic potential.
Therefore, Poisson's equation has to be replaced by Maxwell's equations 3, 4 - 5 for the electromagnetic field. ¿From the mathematical point of view this means that the elliptic Poisson equation is replaced by a hyperbolic system, which complicates in particular the problem of uniqueness and regularity.
Global existence of weak solutions $(\rho, \mathbf{E}, \mathbf{H})$ to problem 1 - 5, with

$$\rho \in L^2_{loc}([0,\infty), H^1(\Omega)) \cap L^\infty_{loc}([0,\infty), L^\infty(\Omega)) \cap C([0,\infty), L^2(\Omega))$$

and $(\mathbf{E}, \mathbf{H}) \in C([0,\infty), L^2(\Omega))$ is proved, see [6] and [9].
Uniqueness and L^p-regularity $(p > 2)$ of weak solutions in the two-dimensional case is proved in [7].
The main question of this talk is the asymptotic behavior of weak solutions for $t \to \infty$. For this purpose assume that $\varphi \in H^1(\Omega)$ and $r_1, r_2 \in L^\infty(\Omega) \cap H^1(\Omega)$ is a solution of the stationary drift-diffusion-equations. Then sufficient conditions for the convergence of $\rho, \mathbf{E}, \mathbf{h}$ to the stationary state r, φ,
i.e. $\lim_{t\to\infty} ||\mathbf{E}(t) + \nabla\varphi||_{L^2(\Omega)} = 0$ and $\lim_{t\to\infty} ||\rho(t) - r||_{L^p(\Omega)} = 0$ for all $p \in [1,\infty)$ will be given.

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