A version of Pohožaev's non-existence result via maximum principles*

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In 1965, S.I. Pohožaev discovered an important identity: if $\Omega \subset \mathbb{R}^N$ is a bounded, smooth domain and if $u \in C^2(\Omega) \cap C^1(\overline{\Omega})$ satisfies

$$\Delta u + f(u) = 0 \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega \tag{1}$$

then

$$\int_{\partial\Omega} \frac{1}{2}(\nabla u)^2 x \cdot \nu \, d\sigma = \int_{\partial\Omega} NF(u) + \frac{2-N}{2} f(u)u \, dx, \tag{2}$$

where ν is the exterior normal and $F(s) = \int_0^s f(t)\,dt$. The proof is by integration by parts. An important application is non-existence of positive solutions of (1), if Ω is strictly star-shaped and

$$F(s)/s^{\frac{2N}{N-2}} \text{ is nondecreasing in } s \in (0, \infty). \tag{3}$$

We prove non-existence via maximum principles under the assumption that

$$f(s)/s^{\frac{N+2}{N-2}} \text{ is nondecreasing in } s \in (0, \infty). \tag{4}$$

Similar non-existence conditions for non-autonomous problems $\Delta u + g(x, u) = 0$ can also be formulated. Condition (4) implies (3), but our approach needs less regularity of Ω and u, and it sheds new light on the supercriticality and star-shapedness assumptions.

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