Existence results for a class of degenerate elliptic equations

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We investigate the existence of positive solutions to Dirichlet problems of the form

$$-\mathrm{div}(a(x)\nabla u) = f(x,u) \text{ in } \Omega,$$

where Ω is a domain in $\mathbb{R}^n$, a is a non negative function which vanishes somewhere and may be unbounded at infinity, and f is a smooth function such that $sf(x,s) \sim |s|^p$ as $|s| \to \infty$, being $p > 1$. According to the behavior of a at its zeroes and at infinity, we can distinguish a subcritical case and a critical case, and in both situations we obtain some existence results. In particular, we study the critical case for the model equation

$$-\mathrm{div}(|x|\alpha\nabla u) = |u|^{2\alpha^*-2}u,$$

where $2\alpha^* = \frac{2n}{n-2+\alpha}$, and we prove that a positive ground state solution exists for the corresponding Dirichlet problem on a domain Ω whenever Ω is a cone, including the case $\Omega = \mathbb{R}^n$. Moreover, concerning the case of an arbitrary domain Ω, we show how the geometry of Ω near the origin and at infinity affects the existence or non existence of ground state solutions.