Attractors for families of processes in weak topologies of Banach spaces

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Processes are particularly useful in the study of evolution PDE's; it is well-known that for autonomous PDE's the long-time behavior of the solution may be described by a family of operators which enjoy the semigroup properties: processes represent the generalization of semigroups to the nonautonomous case. In the last few years, the attention has turned to families of processes. In order to apply the tools involved in these theories some compactness properties of the process defined by the equation are requested; even for fully nonlinear PDE's, when the equation is dissipative, standard energy estimates usually suggest to study the long-time behavior of the solution in the norm topology of some Banach space or with the metric of some nonlinear space. However, in some cases the nonlinearity in the equation only allows to prove nice compactness properties of the corresponding process in the weak topology; a general theory which also allows to treat families of processes for these equations seems not to be known: here, we make an attempt in this direction. To this end, we introduce a self-contained analysis which is based on the notion of "uniformly compactifying at infinity" family of processes: this notion allows to study the compactness properties of the problem in terms of sequences. Our results concern the case of weak topologies: assuming that the family of processes is uniformly compactifying at infinity, we reach sufficient conditions for the existence and connectedness of attractors when the phase space is some reflexive separable Banach space endowed with its weak topology. In literature, the study of evolution equations in the weak topology is usually handled in a different way: one first proves the existence of an absorbing set which is bounded in the norm topology and hence, relatively compact in the weak topology; next one uses the metric induced by the weak topology on bounded sets. The aim of our work is to insert these problems into a more general framework and to avoid the use of a metric. We apply our results to the study of the long-time behavior of the solutions of some nonautonomous parabolic equations when the forcing term is free to run in a suitable bounded subset of a functional space.

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