Global solutions to some reaction diffusion systems with a balance law

M. KIRANE
Universite de Picardie Jules Verne
60 000, Beauvais
France
Mokhtar.kirane@aub.u-picardie.fr

We obtained results on existence and large time behaviour of global classical nonnegative solutions of

$$\begin{aligned} u_t - a\Delta u - b\Delta v &= -f(u)g(v) \qquad \text{in} \qquad \mathbf{R}^+ \times \Omega & (0.1)\\ v_t - c\Delta u - d\Delta v &= +f(u)g(v) \qquad \text{in} \qquad \mathbf{R}^+ \times \Omega & (0.2) \end{aligned}$$

with boundary and initial conditions

$$\frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0 \qquad \text{on} \qquad \mathbf{R}^+ \times S \tag{0.3}$$

$$0 \leq u_0(x) \leq M := \max_{x\in\Omega} u_0(x), \qquad 0 \leq v_0(x) \leq N := \max_{x\in\Omega} v_0(x). \tag{0.4}$$

The nonlinearities $f(u)$ and $g(v)$ are nonnegative differentiable functions, $f(0) = 0$.

Example: The Frank-Kamenetskii approximation to an m^{th}-order exothermic chemical reaction of Arrhenius type where the reaction term is

$$f(u)g(v) = u^m e^v \text{ with } m \geq 1.$$

Results

1. **Case of lower-triangular matrices :** $b = 0$.

Theorem 1. There exists a unique nonnegative global bounded solution $(u(t,x), v(t,x))$ of problem $(1)-(4)$ for **any** nonlinearities $f(u)$ and $g(v)$ whenever
$a > d, \qquad c \geq 0 \qquad$ and $\qquad v_0(x) \geq (c/(a-d))u_0(x)$.

Moreover
$(u(t,x), v(t,x)) \longrightarrow (k_1, k_2) \qquad$ as $\qquad t \to +\infty$ uniformly in Ω,
where

$$k_1 \geq 0,\ \ k_2 \geq 0,\ \ f(k_1)g(k_2) = 0, \qquad k_1 + k_2 = \frac{1}{|\Omega|}\int_\Omega (u_0(x) + v_0(x))\,dx.$$

Theorem 2. Assume that

1. $f(v)$ is bounded in a neighbourhood of $v = -\infty$, and
2. $c \geq d - a > 0$

hold true. Then there exists a unique global solution $(u(t,x), v(t,x))$ of problem $(1)-(4)$.

2. **Case of a full matrix.**

Notation.
$N1.$ $\quad \gamma_1 := \left(d - a - \sqrt{(d-a)^2 + 4bc}\right)/2b.$
$N2.$ $\quad \gamma_2 := \left(d - a + \sqrt{(d-a)^2 + 4bc}\right)/2b.$

Theorem 3. Assume that

$H1.$ $\quad w_{10}(x) := w_1(0,x) = v_0(x) - \gamma_1 u_0(x) \geq 0;$
$H2.$ $\quad w_{20}(x) := w_2(0,x) = v_0(x) - \gamma_2 u_0(x) \geq 0;$
$H3.$ $\quad c > d - a > 0, \qquad 0 < b << 1;$
$H4.$ $\quad f(v) = v^n,\ \ n \geq 1$ is an odd integer;
and
$H5.$ $\quad |d + b - a - c| < (\gamma_1 + 1)/(\gamma_1 C_p).$

hold true. Then there exists a unique global solution to problem $(1)-(4)$.