Blow up patterns in semilinear heat equations with supercritical exponent

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We consider the semilinear heat equation

$$\begin{cases} u_t - \Delta u = u^p, \ x \in \Omega, \ \ t \in [0,T], \\ u(t,x) = 0, \ x \in \partial\Omega, \ \ t \in [0,T], \\ u(0,x) = u_0(x), \ x \in \Omega, \end{cases} \tag{1}$$

where Ω is a ball in $\mathbb{R}^N$ or $\Omega = \mathbb{R}^N$, $u_0 \in L^\infty(\Omega)$ is continuous, nonnegative and radially symmetric. We assume in addition that p is supercritical, $p > \frac{N+2}{N-2}$ and $N \geq 3$. We study the blow up behavior of u solution of (1) around its singularity $x = 0$ as the blow up time is approached. We prove that there is a class of solutions of (1), with Ω a ball in $\mathbb{R}^N$, which blow up at $t = T_m < +\infty$, $x = 0$, and are such that either u blows up faster than the self-similar solutions, that is

$$\lim_{t\uparrow T_m} (T_m - t)^{\frac{1}{p-1}} \|u(t)\|_{L^\infty(\Omega)} = +\infty,$$

or u has a nontrivial self-similar blow up pattern around $x = 0$, that is

$$\lim_{t\uparrow T_m} (T_m - t)^{\frac{1}{p-1}} u(t, (T_m - t)^{1/2} y) = \varphi(y) \ \text{ uniformly on } \ |y| \leq K, \ K > 0,$$

where φ admits the following asymptotic expansion

$$\varphi(y) = C|y|^{-\frac{2}{p-1}}\{1 + o(1)\}, \ \text{as } |y| \to +\infty,$$

with $C > 0$.