Existence, decay and symmetry of solutions of hamiltonian systems in $\mathbb{R}^N$

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We study the existence and the decay of solutions of systems of the type

$$\left| \begin{array}{rcl} -\Delta u + u & = & g(r,v) \\ -\Delta v + v & = & f(r,u) \end{array} \right. \tag{1}$$

in $\mathbb{R}^N$, $N \geq 3$. Using an infinite dimensional linking theorem we show that such a system has a positive radial solution under the standard "mountain pass" assumptions on the nonlinearities, provided their growths at infinity lie below the so-called "critical hyperbola". This result generalises a recent paper by de Figueiredo and Yang ([1]). We also prove their conjecture that the solutions decay at infinity together with their derivatives.

We also study the symmetry of solutions of more general systems in $\mathbb{R}^N$

$$\left\{ \begin{array}{lll} \Delta u_i + f_i(r, u_1, \cdots, u_n) = 0 & \text{in} & \mathbb{R}^N,\ i = 1, \cdots, n, \\ u_i > 0 & \text{in} & \mathbb{R}^N, \\ u_i(x) \to 0 & \text{as} & r = |x| \to \infty, \end{array} \right. \tag{2}$$

where $n \geq 1$, $N \geq 2$ are arbitrary integers. We prove that solutions of (2) are necessarily radial with respect to the same origin provided the system is cooperative, fully coupled and all n-principal minors of $(\frac{\partial f_i}{\partial u_j})$ are nonnegative near zero. This is an extension to $\mathbb{R}^N$ of a well-known result of Troy ([3]) and a generalisation to systems of the symmetry results for single equations ([2]). This result is obtained in a joint work with J. Busca.

References

[1] D.G. de Figueiredo, J. Yang, *Decay, symmetry and existence of positive solutions of semilinear elliptic systems*, To Appear in Nonl. Anal.

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[3] W.C. Troy, *Symmetry properties in systems of semilinear elliptic equations*, J. Diff. Eq. **42** (1981), pp. 400-413.